Ibn KHaldoun-Tiaret University – Department of Physics

Remplacement exam Maths 2 *lasts 1:30 minutes* *Mai 20, 2024*

Solution 1 *1. We have*

$$\begin{aligned}\det M_\alpha &= \det\begin{pmatrix} 1 & 3 & \alpha \\ 2 & -1 & 1 \\ -1 & 1 & 0 \end{pmatrix} \\ &= 1\begin{vmatrix} -1 & 1 \\ 1 & 0 \end{vmatrix} - 3\begin{vmatrix} 2 & 1 \\ -1 & 0 \end{vmatrix} + \alpha\begin{vmatrix} 2 & -1 \\ -1 & 1 \end{vmatrix} \\ &= \alpha - 4 \text{............}\mathbf{\mathit{1.5}}\end{aligned}$$

2. the map f associated with M_α is bijective if and only if $\det M_\alpha \neq 0$ i.e $\alpha \neq 4$........***1pt***

3. since $\det M_\alpha \neq 0$ i.e $\alpha \neq 4$...........***0.5 pt***
then M_α have an anvese M_α^{-1} and

$$\begin{aligned}M_\alpha^{-1} &= \frac{1}{\det M_\alpha}\left(\mathbf{com}M_\alpha\right)^t \text{...........}\mathbf{\mathit{0.5pt}} \\ &= \frac{1}{\alpha-4}\begin{pmatrix} -1 & -1 & 1 \\ \alpha & \alpha & -4 \\ \alpha+3 & 2\alpha-1 & -7 \end{pmatrix}^t \\ &= \frac{1}{\alpha-4}\begin{pmatrix} -1 & \alpha & \alpha+3 \\ -1 & \alpha & 2\alpha-1 \\ 1 & -4 & -7 \end{pmatrix} \text{.........}\mathbf{\mathit{1pt}}\end{aligned}$$

The formula of f_α^{-1}
Let $u(x,y,z) \in \mathbb{R}^3$, we have

$$f^{-1}(x,y) = f^{-1}(xe_1 + ye_2 + ze_3) \text{.......}\mathbf{\mathit{1pt}}$$

since f_α^{-1} is a linear map we deduce that

$$\begin{aligned}f^{-1}(x,y,z) &= xf_\alpha^{-1}(e_1) + yf_\alpha^{-1}(e_2) + zf_\alpha^{-1}(e_3) \\ &= \frac{1}{\alpha-4}\begin{pmatrix} x(-e_1 - e_2 + e_3) + y(-e_1 + \alpha e_2 + \alpha e_3) \\ +z((\alpha+3)e_1 + (2\alpha-1)e_2 - 7e_3) \end{pmatrix} \text{.......}\mathbf{\mathit{1pt}}\end{aligned}$$

then

$$f^{-1}: \begin{array}{c} \mathbb{R}^3 \longrightarrow \mathbb{R}^3 \\ (x,y,z) \longmapsto \frac{1}{\alpha-4}\left(-x+\alpha y+(\alpha+3)z, -x+\alpha y+(2\alpha-1)z, -x-4y-7z\right). \end{array} \text{.......}\mathbf{0.5}$$
pt

Solution 2 *We have Compute the limited development up to order 3 at the neighborhood of 0 for f such that*

$$f(x) = \ln(1 + \tan x).$$

According to the table of limited developments, we have:

$$\ln(1+u) = u - \frac{1}{2}u^2 + \frac{1}{3}u^3 + u^3\varepsilon(u) \,\boldsymbol{.......0.5\ pt}$$

and

$$\tan x = x + \frac{1}{3}x^3 + x^3\varepsilon(x) \,\boldsymbol{.......0.5\ pt}$$

then

$$f(x) = \ln(1+\tan x) = \ln\left(1 + x + \frac{1}{3}x^3 + x^3\varepsilon(x)\right) \boldsymbol{.......0.5\ pt}$$

Substituting $u = x + \frac{1}{3}x^3 + x^3\varepsilon(x)$, $\left(\lim\limits_{x\to 0} u = 0\right)$ ***........0.5pt***
We get Calculating u, u^2, *and* u^3 *up to order* 3 *in* x, *we obtain*

$$\ln(1+\tan x) = x - \frac{1}{2}x^2 + \frac{2}{3}x^3 + x^3\varepsilon(x) \,\mathbf{.......2pt}$$

Solution 3 *Let* $x = a + b - z$ *then* $dx = -dz$***.......0.5pt***

$$\begin{aligned} \int_a^b xf(x)\,dx &= -\int_b^a (a+b-z)\,f(a+b-z)\,dz \boldsymbol{.......0.5pt} \\ &= \int_a^b (a+b-z)\,f(z)\,dz \boldsymbol{......0.5pt} \\ &= (a+b)\int_a^b f(x)\,dx - \int_a^b xf(x)\,dx \boldsymbol{......0.5pt} \end{aligned}$$

then

$$\int_a^b xf(x)\,dx = \frac{a+b}{2}\int_a^b f(x)\,dx \boldsymbol{.......1\ pt}$$

2. The value of

$$I = \int_0^\pi \frac{x\sin x}{1+\cos^2 x}dx.$$

taking $f(x) = \frac{\sin x}{1+\cos^2 x}$***......0.5pt***
and we remark that $\frac{\sin(0+\pi)x}{1+\cos^2(0+\pi)} = \frac{\sin x}{1+\cos^2 x}$ ***......0.5pt***

then

$$\int_0^{\pi} \frac{x\sin x}{1+\cos^2 x}dx = \frac{0+\pi}{2}\int_0^{\pi} \frac{\sin x}{1+\cos^2 x}dx \textbf{......0.5pt}$$
$$= -\frac{\pi}{2}\left[\arctan(\cos x)\right]_0^{\pi} \textbf{......0.5pt}$$
$$= -\frac{\pi}{2}\left(-\frac{\pi}{4}-\frac{\pi}{4}\right) = \frac{\pi^2}{4}\textbf{......1pt}$$

3. We have

$$x\frac{dy}{dx}+y = xy^3 \quad i.e\ xy'+y = xy^3 \tag{1}$$

This is a Bernoulli differentiable equation 1 , where $\alpha = 3$***......0.5pt***
We first divided the equation throught by y^3*, thereby expressing it in the equivalent form*

$$x\frac{y'}{y^3}+\frac{1}{y^2} = x\textbf{......0.5pt.} \tag{2}$$

by using the change variable $z = y^{1-\frac{1}{2}}$*, then* $z' = \frac{1}{2}\frac{y'}{\sqrt{y}}$ ***......0.5pt***
the equation 2 transforms into

$$xz' - 2z = -2x\textbf{......0.5pt} \tag{3}$$

the solution of linear differential equation of 1st order 3 is

$$z = Cx^2 + 2x\textbf{.......0.5pt}$$

Thus we obtain the solutions of **??** *in then form*

$$y = \frac{1}{\sqrt{Cx^2+2x}}\textbf{.......0.5pt}$$